PiXL Independence:

Mathematics - Student Booklet

KS5

Topic 2 – Polynomials

Contents:

I. Basic Skills Check – 10 credits per skill check

II. Short Exam Questions - 30 credits per section

III. Further Practice – 30 credits each

IV. Investigations – 80 credits each

V. Academic Stretch – 50 credits each

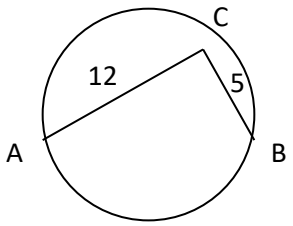
I. Basic Skills Check

Answer the following questions. In order to improve your basic arithmetic you should attempt these without a calculator.

10 credits for completing this quiz.

Skills Check 1

1. Rewrite the equation of the straight line $2x+3y=6$ in the form $y=mx+c$.
2. Factorise $x^2+5x-24$.
3. Sketch the graph of $y=x+5$.
4. Write $\frac{1}{x^3}$ in the form x^n.
5. Show that the lines $2x-5y=10$ and $10y-4x-5=0$ are parallel.
6. Given $f(x)=x^3+3x^2-6x-8$, find the value of $f(2)$.
7. Express $\left(\sqrt{7}+1\right)^2$ in the form $a+b\sqrt{7}$, where a and b are integers.
8. Solve the inequality $5 \langle 2x-1 \langle 17$.
9. Find the length of AB, given X is the centre of the circle.



Skills Check 2

1. Solve the inequality $6(x+3) \rangle 8-2(x+1)$.
2. Sketch the graph of $y=3x^4$.
3. Work out the point of intersection of the two lines $x-2y=5$ and $2x=5y+7$.
4. Simplify $6\sqrt{2}+5\sqrt{8}$.
5. Write down the mid-point of (4, 5) and (6, 3).
6. Write $\frac{1}{4x^3}$ in the form kx^n.
7. Solve the equation $2x^2+x-4=0$, leaving your answer in surd form.
8. It is given that $f(x)=x^3+7x^2+8x+10$. Find the value of $f(1)$ and $f(-1)$.
9. The points A and B have coordinates (12, 5) and (7, 3). Find the gradient of AB.
10. Factorise x^2-9.

Skills Check 3

1. Write down the reciprocal of $\frac{1}{5}$.

2. Given $\frac{3+\sqrt{5}}{4+\sqrt{5}} = p + q\sqrt{5}$, where p and q are rational numbers, find p and q.

3. Sketch the graph of $y = -x^2$.

4. Solve $6x^2 + 11x + 3 = 0$ by factorisation.

5. Solve the inequality $-3 \leq \frac{x}{2} \leq 5$.

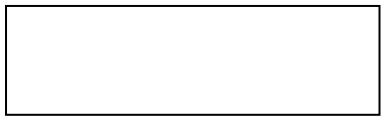
6. Given $P(x) = 2x^3 + x^2 - 4x + 5$, evaluate $P(2)$.

7. Write down the mid-point of (2, 10) and (-3, 0).

8. Write down the gradient of the line joining (2, 10) and (-3, 0).

9. Solve the simultaneous equations $3x + 2y = 6$ and $y = 5x - 10$.

10.



The area of this rectangle is $y$cm^2 and the perimeter is $y$cm. Find the dimensions of the rectangle.

II. Short Exam Questions

Section 1 - Algebraic Division

1. Given that $4x^3 - 25x^2 - 23x + 14 \equiv (x-7)(px^2 + qx + r)$, find the values of the constants p, q and r.

2. You are given that $f(x) = x^3 + x^2 - 14x - 24$,
   a) Write $f(x)$ in the form $(x+2)(ax^2 + bx + c)$.
   b) By first factorising the quadratic part of your answer to (a), express $f(x)$ as a product of three linear factors.

3. You are given that $g(x) = x^3 - 4x^2 - 7x + 10$,
   a) Write $g(x)$ in the form $(x-1)(ax^2 + bx + c)$.
   b) Hence express $g(x)$ as a product of three linear factors.
   c) Hence solve the equation $g(x) = 0$.

4. The polynomial $p(x)$ is defined by $p(x) = 3x^3 - 29x^2 + 62x + 24$
   You are given that $(x-6)$ is a factor of $p(x)$.
   a) Factorise $p(x)$ completely.
   b) Hence simplify $\dfrac{2x^2 - 8x}{3x^3 - 29x^2 + 62x + 24}$.

5. The polynomial $p(x)$ is defined by $p(x) = 2x^3 + 5x^2 + x - 2$.
   Given that $(2x-1)$ is a factor of $p(x)$.
   a) Write $p(x)$ as a product of three linear factors with integer coefficients.
   b) Simplify the algebraic fraction $\dfrac{3x^2 + 6x}{2x^3 + 5x^2 + x - 2}$ as far as possible.

Section 2 - Factor Theorem

1. Given that $f(x) = x^3 - 7x - 6$,
   a) Find f(1), f(-1), f(2), f(-2), f(3) and f(-3).
   b) Hence write $f(x)$ as a product of three linear factors.
   c) Solve the equation $f(x) = 0$.

2. Given that $g(x) = x^3 - 3x^2 - 6x + 8$.
   a) Use the factor theorem to show that (x-1) is a factor of g(x).
   b) Factorise g(x) completely.
   c) Solve the equation g(x) = 0.

3. Given that $h(x) = x^3 - 3x^2 - 16x - 12$.
   a) Use the factor theorem to show that (x+2) is a factor of h(x).
   b) Write h(x) in the form $(x + 2)(x^2 + px + q)$ where p and q are constants to be determined.
   c) Solve the equation h(x) = 0, leaving your answers in surd form where appropriate.

4. Given that $g(x) = x^3 + ax + 6$.
   a) If (x+3) is a factor of g(x), show that a = -7.
   b) Hence solve the equation g(x) = 0, giving answers in **surd form** where appropriate.

5. The function $f(x) = x^3 + Ax^2 + Bx + 10$ has factors (x+2) and (x-5).
   a) Use this information to form and solve two simultaneous equations to find A and B.
   b) Factorise f(x) completely.

Section 3 - Polynomials and Graphs

1. Given that $f(x) = 5x^3 - 6x^2 - 9x + 2$.
   a) Use the factor theorem to show that $(x-2)$ is a factor of $f(x)$.
   b) Solve the equation $f(x) = 0$.
   c) Hence sketch the graph $y = f(x)$, labelling the points where the curve crosses the coordinate axes.

2. You are given that $f(x) = (x+1)(x+2)(x-2)$.
   a) Solve the equation $f(x) = 0$.
   b) What does this tell you about the graph of $y = f(x)$?
   c) Where will the graph cross the y-axis?
   d) Sketch the graph of $y = f(x)$.
   e) Write the equation of the graph in the form $y = ax^3 + bx^2 + cx + d$.

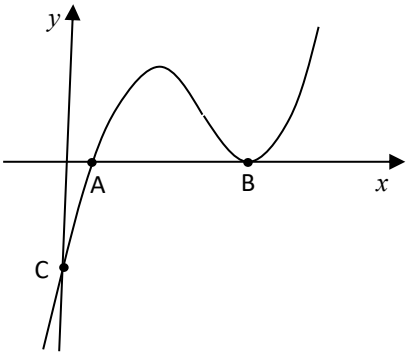
3. Solve the equation of $(x+4)(x-2)(x-1) = 0$.
   a) What does this tell you about the graph of $y = (x+3)(x+2)(x-1)$?
   b) Where will the graph cross the y-axis?
   c) Sketch the graph of $y = (x+4)(x+2)(x-1)$.
   d) Write the equation of the graph in the form $y = ax^3 + bx^2 + cx + d$.

4. The sketch opposite shows the curve $y = (2x-3)(x-5)^2$.

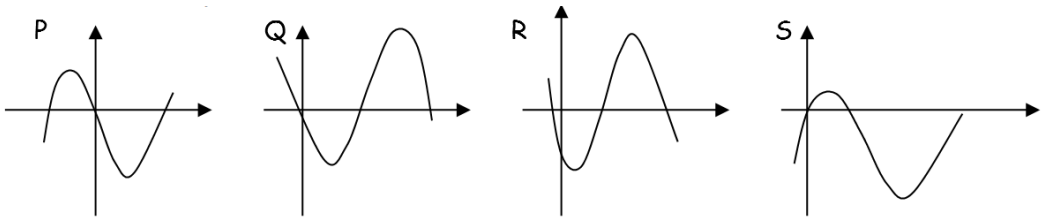
   a) The constants A, B and C on the sketch indicate the points where the curve meets the coordinate axes.

      Write down the values of A, B and C.

   b) Write the equation of the graph in the form $y = ax^3 + bx^2 + cx + d$.



5. Show that $x(x-4)(3-x)$ is equivalent to - $x^3 + 7x^2 - 12x$. Which of the graphs below is a sketch of $y = -x^3 + 7x^2 - 12x$?



Section 4 - Mixed Questions

1. $f(x) = x^3 + (a+1)x^2 - 18x + b$, where a and b are integers.

Given that $(x - 4)$ is a factor of $f(x)$.

(a) Show that $16a + b + 8 = 0$.

Given that $(x + a)$ is also a factor of $f(x)$, and that $a > 0$.

(b) Show that $a^2 + 18a + b = 0$.

(c) Hence find the value of a and the corresponding value of b.

(d) Factorise $f(x)$ completely.

2. The polynomial $P(x) = x^3 - 4x^2 + kx - 4$ leaves a remainder of –2 when divided by $(x-1)$.
   a) Find the value of the constant k.
   b) Show that $(x-2)$ is **not** a factor of $P(x)$.

3. Sketch on a single diagram the following graphs:
   a) $y = x(x+3)(2-x)$.
   b) $y = -\frac{3}{x}$.
   c) Using your sketch, giving reasons, find the number of real solutions to the equation $x(x+3)(2-x) + \frac{3}{x} = 0$.

4. The function $f(x) = x^3 + Ax^2 + Bx - 30$ has factors (x-2) and (x+5).
   a) Use this information to form and solve two simultaneous equations to find A and B.
   b) Factorise f(x) completely.

III. Further Practice

1. **Watch the video and answer all the questions. Try the interactive resource at the bottom of the video. What do you notice about the graphs?**
   https://www.examsolutions.net/tutorials/sketching-quadratic-graphs/?level=A-Level&board=Edexcel&module=C1&topic=1196

2. **Assignment task to complete.**
   http://tutorial.math.lamar.edu/ProblemsNS/Alg/Polynomials.aspx

3. **Create your own worksheet with at least ten polynomial multiplication problems and answer them.**
   http://www.webmath.com/wsheet1.html

4. **Watch the following sections and complete the exam style questions:**
   a) **"Algebraic Long Division"**
   b) **"Factor Theorem"**
   c) **"Sketching Cubic and Reciprocal Curves"**
   https://www.examsolutions.net/as-maths/edexcel/pure-maths-as-tutorials/

IV. Investigations

For each of the following you should carry out the investigations then read the notes. You need to keep a detailed summary of what methods/approaches you have tried and what you then changed each time.

1. **Read the notes on the page and carry out the graph investigation. Complete the worksheet included, there's also an extension task to gain extra credit.**
   https://www.teachmathematics.net/page/14038/exponential-graphs

2. **Euler's 9 Point Circle.** Read the article and follow the instructions in order to create a 9 point circle. Create a document/poster to instruct someone else on how to do this.
   https://ibmathsresources.com/2017/09/30/eulers-9-point-circle/

3. **Read the article, follow some of the links. Can you write a summary of the key points and the maths involved?**

   https://nrich.maths.org/2769

4. **Read the article and the comments. Which arguments are convincing? Are there flaws in the responses? What would your response be if you were to comment on the article?**

   https://plus.maths.org/content/maths-minute-two-envelopes-problem

5. **Risp**
   http://www.s253053503.websitehome.co.uk/risps/risp-8.pdf

V. Academic Reading

1. Read the following lecture notes and make a detailed summary.

   https://www.maths.ox.ac.uk/system/files/attachments/lecture2.pdf

2. Complete STEP assignment 1/read the article to extend your understanding.

   https://maths.org/step/assignments/assignment-1

   https://plus.maths.org/content/ringing-changes

3. Read the following and make a detailed summary.
   https://nrich.maths.org/1422

4. Read the following and conduct your own research to hidden maths in other structures
   https://plus.maths.org/content/maths-minute-st-pauls-dome

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